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A GAME-THEORETIC APPROACH TO THE NEGOTIATION PROBLEM

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
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ABSTRACT

The essential elements of a negotiation are examined. The formulation and solution of basic games are discussed. The negotiation process is formulated as a game in order to demonstrate the value of Game Theory in providing insight into certain aspects of negotiation. The aspects specifically treated include selection of an initial position, cooperation, threats, and coalitions.

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1. Introduction.

This paper is written from the point of view of the Operations Analyst. In particular it attempts to apply the Scientific Method to a subject which abounds in non-quantitative aspects. The Operations Analyst does not provide quantitative solutions to non-quantitative problems. He does, however, provide quantitative insight into such problems. The insight is not intended to be a solution. Instead it is offered merely as one facet of the evidence which the competent executive evaluates in the course of his decision process. On occasion the analysis will yield nothing of value. Often, however, the analysis will privide either affirmation of estimates obtained by other means or contradictions which point to a review of hypotheses.

This paper looks at the Negotiation Process from an analytic point of view. It points out the parallel between certain aspects of Negotiation and Game Theory. To this end a brief discussion of the Negotiation Process is given followed by a brief survey of the Theory of Games. Those aspects of the Negotiation Process which have a particular analog in the Theory of Games include selection of an initial position, co-operation, threats, and coalitions. Game Theory is shown to be a convenient vehicle for viewing these problems in a quantitative frame of reference. A technique is suggested for estimat-

ing the probabilities an opponent has associated with possible courses of action.

2. Prerequisites of a Negotiation

There are certain prerequisites to any negotiation: parties to the negotiation, an issue, and a willingness on the part of the parties to negotiate.

A party to a negotiation is an individual representing himself or some legitimate organization which has a measurable interest in an issue.

A debatable issue must be at the heart of every negotiation. It may be an object such as a manufactured product or art object. An issue may also be a service such as that rendered by domestic help. Finally, an issue may be a situation such as the Nuclear Test Ban Treaty or a defensive alliance. An issue may be simple or complex, but it must be clearly defined and thoroughly understood by all parties to the negotiation.

Willingness to negotiate implies that no one should be involuntarily compelled to negotiate. This restriction rules out such practices as "Dictated Settlements" and so-called "Unconditional Surrender Terms". While these practices exist, they should not be dignified by alluding to them as genuine negotiations. A proper negotiation allows each party the option of declining to negotiate or of resigning from the negotiation at any time. In addition, each party to a negotiation must have at least two feasible alternatives from which to select his

preferred course of action.

3. Elements of a Negotiation

The elements of a negotiation can be conveniently categorized as follows:

1. Research
2. Estimate of other parties' possible positions
3. Formulation of own possible positions
4. Estimate of possible outcomes
5. Selection of tentative initial strategy
6. Confrontation and settlement

As in most disciplines, research is an essential element. The competent negotiator prepares well in advance for his confrontation with other interested parties. An historical investigation would certainly be undertaken in order to trace the issue from its inception to its present status. Of prime importance would be the discovery of any factors which affected the growth or retardation of the issue. In some situations statistics is a valuable tool for presenting large quantities of data in convenient form. Overt and covert intelligence is a fruitful source of information when due regard is given to its inherent uncertainties. A psychological portrait of each other party to the negotiation is of particular value to a negotiator attempting to predict an opponent's response to a specific proposal.

After a factual data base has been established, the next logical step is to formulate an estimate of the possible positions of the other parties to the negotiation. This should be done before one's own positions are formulated, for an effective response must exist to every proposal set forth by the other parties. The importance of this preparedness cannot be overlooked. Considerable psychological advantage can accrue to the negotiator who suddenly announces an extreme or unorthodox proposal that leaves his opponent nonplussed and unable to respond promptly and effectively. The estimates of the other parties' possible positions are generally based on actions in previous negotiations, their publicly stated positions (if any), and partial knowledge of their objectives gleaned from intelligence, observation, etc. The estimates should be complete and include even extreme and apparently unreasonable positions.

The negotiator then proceeds to formulate his own possible positions. In addition to his research and his opponents' possible positions he usually has some sort of policy guidelines established by his own superiors, or the organization he is representing, or himself. This list, like the estimates above, must include not only all reasonable positions but also a set of unorthodox or extreme proposals specifically designed to counter a sudden thrust by one's opponent. After completing this list it must be carefully checked against the estimate of the other parties' possible positions. There must be a respon-

sive counter proposal to each of their possible positions.

Estimating possible outcomes is probably the most difficult preparatory task. Its importance, however, certainly justifies the expenditure of effort. There is no unique method of formulating this estimate, and certainly no optimal method. One technique of some merit is to number all the possible positions of one's opponents in any arbitrary fashion. One's own positions are similarly numbered. The respective number one positions are then compared and the following question is asked:

If the opponents take position number one and I select my position number one, what will the outcome be? Will they stand to gain or lose and by how much? Will I stand to gain or lose and by how much?

Admittedly these are value judgements, and the classic objection to them is their subjectivity and uncertainty. However, we do make value judgements all the time, and the process is acknowledged to be a necessity. Some work has been done in the area of filtering out the bias from value judgements and measuring the uncertainties associated with them. Until the work in this area has proceeded sufficiently, the uncertainty of value judgements will remain a valid criticism of any work employing them. However it seems unnecessarily conservative to suspend any investigation into this area simply because the technique of making value judgements is still uncertain.

The opponents' first position is then compared with all the rest of the negotiator's positions, and a value judgement is made of the mutual outcome for each pair. The process is repeated with all the rest of the opponents' positions until a complete array has been formed showing the estimated outcomes for each possible pair of positions. This array can be conveniently displayed in a matrix whose elements are ordered pairs of real numbers.

The problem of making value judgements, particularly with the use of a linear utility function, is discussed in Appendix A.

The list of possible outcomes is then carefully arranged and studied. The outcomes might be ordered with respect to desirability, feasibility, enforceability, goodheartedness, or intransigence. Based on this ordering, a selection is then made of a tentative initial position. A selection made in such a manner might now reasonably be called a strategy. It is more than a random selection and in some sense represents a function of the expected gain. There are several options available in the selection of an opening position. Broadly they might be classified as a mild position, a moderate position, or an intransigent position. The mild position and intransigent positions both have a cumbersome drawback in that they only allow for subsequent adjustment in one direction. Prudence

would seem to dictate a position that allows for maximum freedom of movement in the initial stages of the negotiation.

The Theory of Utility, briefly mentioned above, and more fully discussed in Appendix A, is a meaningful technique for estimating possible outcomes. The Theory of Games, on the other hand, has its principal application in the area of selecting initial strategies and in the confrontation and settlement. Such concepts as competition, threats, coalitions, cooperation, the doublecross etc. can be meaningfully looked at with the use of a Game-Theoretic Model.

4. Formulation and Solution of Basic Games.

The simplest form of a Game is the discrete two person zero-sum game with a saddle point. Consider the following array, commonly called a matrix of payoffs. The elements of the matrix represent the units of utility associated with each pair of outcomes.

	Y_1	Y_2	Y_3
X_1	6	1	4
X_2	2	0	5
X_3	4	-1	0

Let the rows represent the possible alternatives available to Player X and the columns represent a set of similar alternatives available to Player Y. In the zero-sum game the elements

of the matrix represent the utility of the payment to Player X by Player Y. Both players must announce their selections at the same time, precluding any particular advantage accruing to either player. Since an element of uncertainty exists, the question naturally arises as to whether or not there is a way of playing this game that is optimal, in some sense, regardless of how one's opponent plays. Let us look at this problem from Player X's point of view.

Note that if X selects alternative 3 he stands to gain 4, 0, or -1 depending on Y's selection. Likewise if X selects alternative 2 his gain can vary from 0 to 5. However, alternative 1 is unique in that the minimum gain is 1 unit of utility regardless of Y's selection. Therefore by playing alternative 1 each time X can guarantee himself a profit of at least 1 unit of utility. In this sense alternative 1 is optimal and the decision to play it all the time can properly be called a strategy. A similar examination of the outcomes from Y's point of view reveals that Y can insure himself against losing more than 1 unit of utility by always playing alternative 2. In the same sense, the decision by Y to always select alternative 2 in order to minimize his losses can properly be called strategy. The payoff 1 is unique in that it is simultaneously the minimum of its row and the maximum of its column. Such an element is called a Saddle Point. A number that is simultaneously the minimum that X can expect to win and the maximum

that Y can expect to lose is called the Value of the game. In games having a saddle point, the saddle point is always the value of the game. Selection of alternatives which yield the value of the game is called an optimal strategy. If the selection is invariably restricted to a single alternative, as in the game above, then the strategies are called pure strategies.

Some games may have more than one saddle point, as in the following game:

	Y_1	Y_2	Y_3
X_1	-2	2	-4
X_2	2	2	3
X_3	-5	1	6

Notice that elements (2,1) and (2,2) are both saddle points. Naturally X selects alternative 2 every time and as such is playing a pure strategy. Y, however, has a choice. Both 1 and 2 are equally good selections. Therefore Y may play 1 all the time or 2 all the time or any probability distribution over the two alternatives. Such a mixture of alternatives is called a Mixed Strategy. No mixed strategy is available to X in this game. It can easily be demonstrated that the intersection of the rows and columns of any two saddle points is itself a saddle point. The proof follows directly from

the definition of saddle point.

A more interesting game, and one more akin to a simple negotiation, is the two person zero-sum game without a saddle point. Consider the following game:

	Y_1	Y_2
X_1	1	2
X_2	3	0

For simplicity each player is assumed to have but two alternatives. No loss of generality is incurred by such a limitation and a great deal of clarity can be preserved. Clearly this game does not have a saddle point, therefore the value of the game, if one exists, is not obvious and the formulation of a strategy is not meaningful. Fortunately a value for this game does exist. The proof of its existence here, and in every game of this form, is contained in the Minimax Theorem first proposed and proved by J. von Neumann and O. Morgenstern in their famous treatise, "The Theory of Games and Economic Behavior" [11]. The proof is complex and the interested reader is referred to the literature for a thorough treatment. The existence of a value for this game leads one to believe that there must exist some strategy that hopefully will yield the value of the game regardless of the selection made by one's opponent. Therefore if V is the value of the game, and if X_1, X_2, Y_1, Y_2 are the

relative frequencies with which the various selections are made, then it must follow that:

$$X_1 (1) + X_2 (3) = V \text{ if Y selects } Y_1$$

$$X_1 (2) + X_2 (0) = V \text{ if Y selects } Y_2$$

$$X_1 + X_2 = 1 \text{ by definition}$$

This is a simple set of simultaneous equations which has a unique solution using either Gauss' rule or Cramer's rule. Larger matrices can be solved using Linear Programming Methods which are amenable to computer techniques. An analysis of the play of the game from Y's point of view yields a similar set of equations. The solution of the simple game described is clearly:

$$X_1 = 3/4,\ X_2 = 1/4 \text{ or } X = (3/4,\ 1/4)\ ,$$

$$Y_1 = 1/2,\ Y_2 = 1/2 \text{ or } Y = (1/2,\ 1/2)\ ,$$

$$V = 3/2\ .$$

Therefore if X plays his first alternative 3/4 of the time and his second alternative 1/4 of the time he can expect to win 3/2 units of utility over the long run. Likewise if Y plays each of his alternatives half of the time he can expect to lose 3/2 units of utility over the long run. The term expectation is important in this form of game. Note that on no single play of the game can X win precisely 3/2 units of utility. The

concept of a value for a game without a saddle point embraces the notion of expected value. If the game is played an infinite number of times X's average winnings will approach 3/2 as a limit, assuming he plays the game according to the optimal probability distribution over his set of alternatives. The same expectation holds true for Y. Note the distinctive difference between this game and the game with a saddle point. In the game with a saddle point X is assured of winning precisely the value of the game on each play if he plays his optimal strategy. In the game without a saddle point X is only assured that his average winnings over a long period of time will approach the value of the game. On any particular play of the game X might win as much as 3 units of utility or as little as zero. Therefore his strategy seems reasonable over a long period of time.

But what meaning does it have if the game is to be played but once? One possible way of utilizing such a strategy in a one-play game is to creat some random number device to determine which alternative will be selected. This is well and good if your opponent does the same thing. Will he? There is no categorical answer except to assume that he will if he is a rational person. Presumably he can perceive the difficulty as clearly as you can and the presumption of rationality is reasonable although not certain. This presumption of rationality is crucial to Game Theory and is stated more precisely

below. It is mentioned here to show its applicability.

In order to look at a non-trivial negotation from a game-theoretic point of view we need a somwhat more sophisticated form of a game than the one previously discussed. This is the 2-person non-zero-sum game. If differs from the zero-sum game in that the elements of the payoff matrix are ordered pairs of real numbers, the first number representing the payoff to the first player and the second number representing the payoff to the second player. The second number is not necessarily the negative of the first. If it is, we have the zero-sum game as a special case.

The non-trivial negotation usually offers several alternatives which are mutually advantageous or mutually damaging, although the advantage or damage need not be the same for each negotiator.

Consider the following simple game:

	Y_1	Y_2
X_1	(1,4)	(7,6)
X_2	(4,20)	(2,3)

Again each player is presumed for simplicity to have but two alternatives. With the introduction of two payoffs in terms of units of utility it is meaningful to point out that the two players need not use the same utility scale. In the case of

a negotiation it is likely that protagonists will utilize different scales. It suffices that ratios of utilities be invariant under a transformation. There are two approaches to solving a game of this sort.

Assumption A. When both players are using the same utility scale.

Assumption B. When each player is using a different utility scale.

Solution under Assumption A.

If both players are using the same utility scale it is appealing to look at the matrix of relative advantages. Our example would translate into a zero-sum matrix of the form:

	Y_1	Y_2
X_1	-3	1
X_2	-16	-1

The non-zero-sum game has now been reduced to the more familiar zero-sum game. Our example, in fact, has a saddle point. Therefore, (1,0) is an optimal strategy for both players in the sense that the relative difference of utility is minimized. Granted that X is in a somewhat unfavorable position to begin with, the selection of the above strategies at least minimizes the bad situation.

If no saddle point exists in the Matrix of Relative

Advantage then the optimial solution to the game is that probability distribution which yields the value of the game to both players.

Solution under Assumption B.

The solution under Assumption B is somewhat less appealing due to the uncertainty about one's opponent's utility scale. The units of utility that appear as the payoff to one's opponent can be either revealed to you by your opponent with no reference to his scale, or estimated by you with an associated probability of exactness. In either case considerable uncertainty exists concerning the implication of any one payoff. Only the assumption of linearity of utility is sustaining in that it makes ratios of utilities meaningful regardless of the scale employed. Solutions under Assumption B require somewhat more detailed treatment because of the uncertainties involved and because opponents in actual negotiations find themselves confronted with Assumption B far more often than Assumption A.

It is helpful to view a non-zero-sum game under this assumption from a geometric point of view. Plotting X's utilities on the abscissa and Y's utilities on the ordinate, we obtain a representation of the following form:

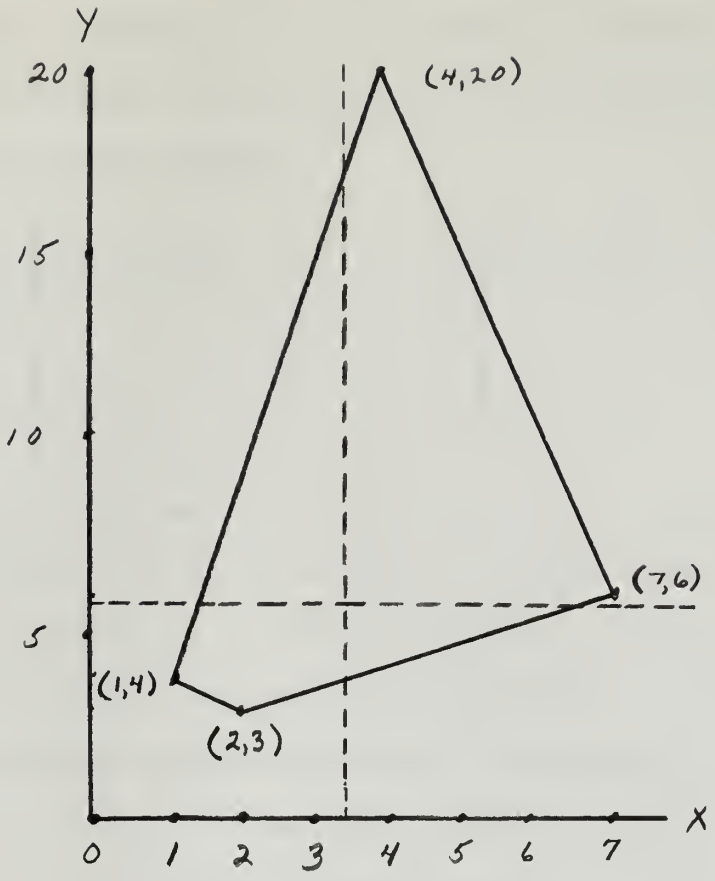


Figure 1

Since the utility scales are related by some unknown linear transformation, the scaling of the ordinate relative to the abscissa is arbitrary. The smallest convex set covering the pairs of payoffs represents the set of all possible outcomes for any combination of probability distributions. Further, because of the unknown, or uncertain, relationship between pairs of payoffs, a matrix of relative advantage is not meaningful. One solution, proposed by Luce and Raiffa [5], Braithwaite [2], Rapoport [7] and many others, separates the

non-zero-sum game into two zero-sum games. In this approach X looks at his matrix of possible payoffs and computes his minimax strategy. Y does the same. Viewed separately the zero-sum matrices become:

	Y_1	Y_2
X_1	1	7
X_2	4	2

	Y_1	Y_2
X_1	4	6
X_2	20	3

Employing the Minimax Criterion X discovers his optimal strategy to be (1/4,3/4), yielding an expected payoff of 13/4 units of utility according to X's scale of reference. Likewise Y discovers his optimal strategy to be (3/19,16/19), yielding an expected payoff of 108/19 units of utility according to Y's scale of reference. These expected payoffs are shown as dotted lines in our geometric representation. In a game played under this assumption the concept of a value to the game is meaningful only when compared with some scale of reference. Since there are two scales of reference employed there are necessarily two values of the game.

5. Assumptions Required in order to Formulate a Negotiation as a Game.

In order to formulate a negotiation as a game the following assumptions must be made:

1) The outcomes of a negotiation must be of such a nature

that they can be measured by a linear utility function. The utility function need not be expressible in algebraic form, but it must have certain linearity properties that are more fully discussed in Appendix A.

2) It must be assumed that the opponent will act rationally. This assumption does not rule out a threat or an unorthodox move. It simply assumes that the opponent has an overall objective and that he intends to maximize it in some sense.

3) It is assumed that a negotiator can evaluate each course of action selected by his opponent and match some meaningful response to it.

6. Selection of an Initial Position.

As previously mentioned the negotiator has a continuum of possibilities from which to select his opening position. This continuum may be broadly reduced to three possibilities, a mild position, a moderate position, or an intransigent position. A moderate position seems preferable initially for the following reasons:

A. It represents the negotiator as a not unreasonable person. Accordingly it imparts some assurance of rationality and reliability.

B. It belies the assumption that the negotiator may be gullible.

C. It permits maximum flexibility in that it allows the

negotiator to subsequently soften or harden his position without appearing particularly weak or overbearing.

D. It provides for a minimumly acceptable payoff even if one's opponent acts irrationally.

E. It allows one to resign from a possibly fruitless negotiation without appearing to surrender.

Refer to the game discussed in the previous section under Assumption B. Note how the Minimax Criterion develops a strategy that yields a position satisfying the requirements above. The Minimax Criterion minimizes both the uncertainties concerning one's opponent's utility scale and his possible irrationality. It provides flexibility in that one can moderate or harden his position, and permit his opponent to do so, while remaining within the convex set of possible outcomes. It provides a hedge against a trap, which imparts a certain degree of comfort to the negotiator while he is in the process of feeling out his opponent. Finally, the selection of an initial position by the Minimax Criterion communicates a certain amount of information in a meaningful, albeit indirect way. It tells your opponent something about yourself and implicity invites him to respond. This tacit method of suggesting some exchange of information opens the door to the possibility of cooperation. The imputation of reasonableness may prompt other parties to the negotiation to consider the possibility of coalitions.

7. Exploring the Possibility of Cooperation.

The Minimax Criterion is admittedly unsatisfactory in some respects, particularly when a game is to be played but once. It yields an expected payoff which is independent of one's opponent's course of action. Suppose, however, that one's opponent's course of action is somehow predictable or controlled. Then the possibility arises that both parties to the negotiation might arrive at a satisfactory agreement that provides for the enhancement of each participant's expected gain. Game Theory imparts a certain amount of quantitative insight into this problem.

Condider the following game, which has numerous variations, commonly called the "Battle of the Sexes". It might equally well apply to two people simultaneously trying to go through a single door or two trucks trying to cross a one-lane bridge from opposite directions. As the "Battle of the Sexes" the situation, briefly, is as follows:

Both spouses, X and Y, want to go out on a particular evening. X prefers to go to the boxing matches. Y prefers to go to the ballet. Neither spouse despises the other's choice but each prefers his own. Assume a certain amount of communication has taken place so that each person is well acquainted with his spouse's utility scale.

The matrix of payoffs might be formulated as follows:

	Y_1	Y_2
X_1	(2,1)	(-1,-1)
X_2	(-1,-1)	(1,2)

Alternative 1 is: Go to the boxing matches.

Alternative 2 is: Go to the ballet.

The solution of this game under Assumption A leads to a matrix of relative advantage of the form:

	Y_1	Y_2
X_1	1	0
X_2	0	-1

(X_1, Y_2) is a saddle point which implies that from a purely competitive point of view each partner should act in total disregard of his spouse. This is not particularly appealing, and one might well suggest that both parties resign from the negotiation and stay home.

Let us now view the game under Assumption B from a geometric point of view.

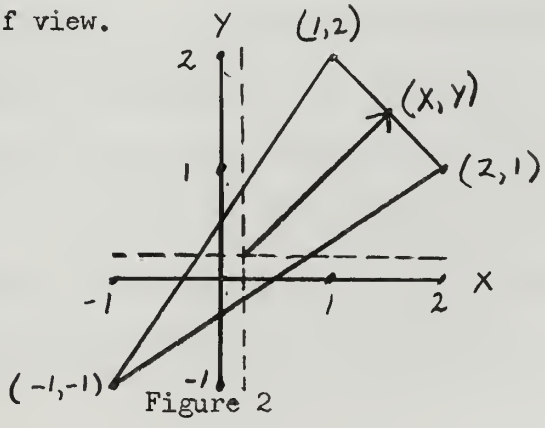


Figure 2

Separating the matrix of payoffs and computing the minimax strategy for each spouse shows that X can gain an expected utility of 1/5 units by selecting the strategy (2/5, 3/5) and Y can gain the same expected payoff by selecting (3/5, 2/5). The selection of a mixed strategy at least offers a more promising mutual advantage than the pure strategy centered on the saddle point.

The situation, although improved, is still far from satisfactory. If both players use their mixed strategies to select their positions, the undesirable outcomes occur 13/25 of the time while the desirable outcomes occur only 12/25 of the time. Therefore if the game is to be played but once, there is a discouragingly high probability that a mutually undesirable outcome will occur.

Assume now that the partners agree to cooperate in some way in order to enhance their prospects of a desirable outcome. Game-Theoretically this is equivalent to translating the value of the game out along a 45 degree line from the Minimax value to the limit of the convex set of possible outcomes, (X,Y). If the partners can move the value of the game to the limit of the set, (X,Y), then they could each realize an expected gain of 3/2 units of utility. This is equivalent to both partners joining forces and playing a reformulated game against nature. To reach the maximum expected value the undesirable outcomes

must be precluded, therefore the reformulated game yielding the maximum expected value is of the form:

	1/2	1/2
(X,Y)	(2,1)	(1,2)

X and Y are now allied and play a game against nature. The selection is made by some random device, perhaps the flip of a coin. X's expectation is now 2 (1/2) plus 1 (1/2) = 3/2. Y's expectation is now 1 (1/2) plus 2 (1/2) = 3/2. Selection of some intermediate outcome between (1/5, 1/5) and (3/2, 3/2) can be effected by both partners playing a joint game against nature of the form:

	P_1	P_2	P_3	P_4
(X,Y)	(2,1)	(1,2)	(-1,-1)	(-1,-1)

The partners agree on a mutually desirable outcome and assign appropriate probabilities to the P's subject to:

$$\sum_{i=1}^{4} P_i X_i = \sum_{i=1}^{4} P_i Y_i$$

The desired outcome must necessarily lie within the convex set and

$$\sum_{i=1}^{4} P_i = 1$$

Cooperation naturally removes much of the competitive aspect from negotiation. Each participant surrenders some of his prerogatives by agreeing to pool his expected gain with his opponent. The circumstances surrounding the negotiation

might well preclude such an arrangement, but when an opportunity for cooperation exists, and repeated efforts at competitive bargaining have produced unsatisfactory results, then a cooperative solution seems to be an obvious exit from stalemate or resignation.

8. Threat Communication and Response.

A threat is a statement by a person to take some particular course of action in order to restrict the opponent's possible responses to a set which that person considers acceptable. The effectiveness of a threat lies wholly in its credibility. If your opponent misunderstands the threat, or doubts your mettle, the gambit is futile and both participants face possible ruin. The diplomatic attitude known as "Brinkmanship" is of this form. It announces an absolute requirement for a certain arrangement and delineates the mutual ruin which will ensure if the arrangement is not met. If your opponent finds the threat incredible and declines to cooperate, you can either crawl shamefully away from the brink or call down mutual disaster by carrying out the threat. Clearly the threat must be communicated effectively and used only sparingly.

From a game-theoretic point of view the threat is an announced decision to select a fixed alternative regardless of the consequences. Unlike the pure strategy which selects a particular alternative because it guarantees some minimum gain regardless of the opponent's choice, the threat is a

selection of a particular alternative with the objective of constraining the opponent to respond in a predetermined manner.

Consider the following game:

	Y_1	Y_2
X_1	(4,1)	(-5,-5)
X_2	(-2,6)	(0,3)

Under Assumption A a saddle point exists in the Matrix of relative advantage at (X_1, Y_2). However if -5 means a loss of 5 million dollars the concept of relative advantages leaves much to be desired.

Under Assumption B, X has an expectation of -10/11 employing the mixed strategy (2/11, 9/11), while Y has an expectation of 1 employing the pure strategy (1, 0). Clearly, then, the game is unfavorable to X. Suppose, therefore that X peremptorily announces that he is selecting alternative X_1 and that no possibility exists of his changing his mind. Y, of course, could acquiesce and also select alternative 1. In fact Y will probably acquiesce if:

A. The threat is clear and credible.

B. X has so formulated the threat that Y's acceptance will yield a payoff greater than the payoff he might expect by dismissing the threat.

Notice also that the game formulated above has a safety

valve which provides a convenient escape in case the threat is misunderstood or deemed incredible. Should Y select Y_2 then X can retreat comfortably to X_2, providing both participants with an enhanced value.

Thus a threat is a worthwhile gambit only if it satisfies conditions A and B and provides some sort of face-saving recourse to both parties.

9. Coalitions.

In many negotiations the number of parties involved is more than two. The natural formulation of this situation in Game Theory would be an n-person non-zero-sum game. In addition to the difficulties inherent in solutions to any non-zero-sum game, we introduce complications, as n gets large, that may make the game unsolvable without making further assumptions about the participants.

One such assumption that is generally made in n-person game theory, if one desires a solution to the game, is that some group of players will correlate their strategies if they can improve their respective expected payoffs by so doing. This procedure is the formation of a coalition. In the theory, formation of coalitions is considered qualitatively outside the formal game. Explicit rules covering possible and forbidden coalitions are not available and have to be considered in the context of the real situation which the game represents. If all of the players are rational in the sense that they wish

to maximize their expected payoff, an additional assumption may be warranted. This is that if a coalition has been formed, the unaffiliated players will form a second coalition since they can do no worse, and may do better, than if they play individually against the strategy of the first coalition. This assumption permits an n-person game to be reduced to a 2-person game and facilitates the mathematical solution.

Unfortunately, the assumption does not appear to be justified for some types of negotiations; for example, negotiations between national governments. The difficulty would seem to be that it is probably impossible to obtain a payoff function which represents the complex interrelationships that exist between nations, especially when the number of nations under consideration is large. An example where the assumption may be justified is a Labor-Management collective bargaining situation where national union formations have been followed by industry-wide associations of management. Here the relationship between the participants, say union locals and individual companies, is repeated many times over and thus has a natural coalition formation structure in an n-person game.

Later we use the above assumption and consider negotiations to be represented by a two-person non-zero-sum game. Any implications to be drawn from the results of this simplifying assumption should be qualified by the considerations outlined in the previous paragraph.

As an example of the formations of coalitions, consider the following payoff matrix representing a three-person non-zero-sum game:

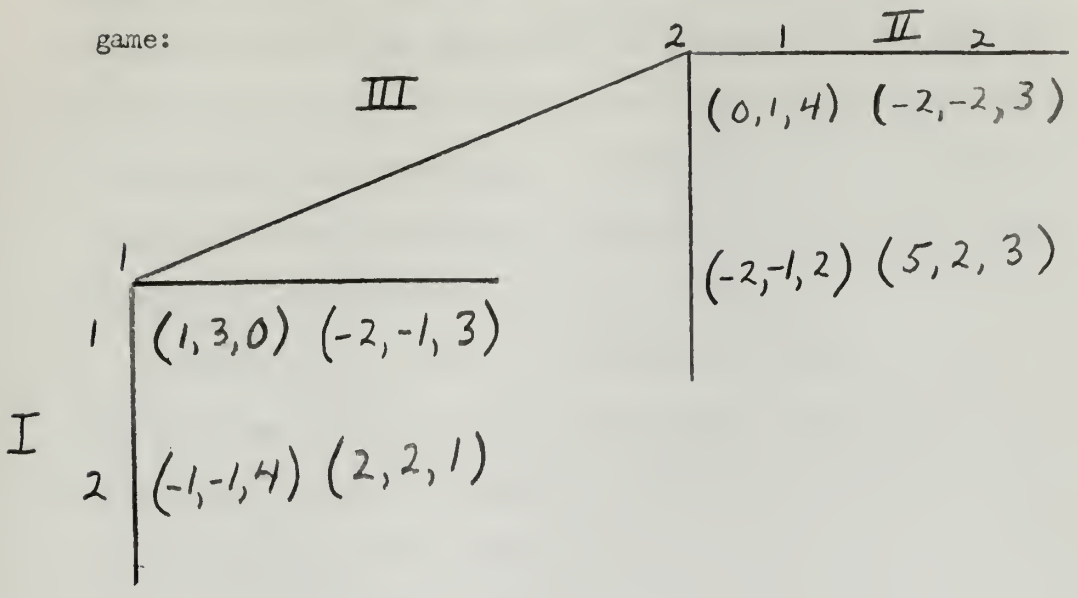


Figure 3

Player I is guaranteed a minimum of -2 no matter which pure strategy he chooses. Player II can guarantee himself a minimum of -1 by playing pure strategy 1 and player III can guarantee himself a minimum of 2 by playing pure strategy 2. These payoffs are the security levels for the respective players.

Now suppose the players use mixed strategies. Players I, II, and III play pure strategy 1 with probabilities p, q and r, respectively. Then the following hold:

$$V_1 = p[-3qr - 7 + 9q + r] + 5 - 7q - 3r + 6qr \,,$$

$$V_2 = q[pr - 3 - 6p] + 2 - 4p + pr \,,$$

$$V_3 = r[-8pr + 6p - 2] + 2pq - p + 3 \,,$$

where V_i is the expected payoff to the ith player.

Obviously, none of the players has much chance of influencing, by himself, his expected payoff. For each player there are too many terms in the expression for his expected payoff over which he has no direct control. If we permit the players to consult with each other prior to the play of the game and arrange to correlate strategies, that is, form a coalition, the expected payoff may be considerably altered.

Suppose we are player I and wish to determine which of players II and III, or both, we could make arrangements with to improve our expected payoff. Consider first that we play pure strategy 1, i.e., $p = 1$. Then

$$\begin{aligned} V_1 &= 3qr - 2 + 2q - 2r \\ &= q[3r + 2] - 2 - 2r \\ &= r[3q - 2] - 2 + 2q . \end{aligned}$$

If we choose to consult with player II, then $0 \leq q \leq 1$. If $q = 1$, then

$$V_1 = r ,$$

which implies that $0 \leq V_1 \leq 1$, which is better than our security level of -2. Also for player II, $V_2 = 1 + 2r$ and hence $1 \leq V_2 \leq 3$ and player II also does better that his security level of -1. This is then a good coalition from the point of view of both player I and player II, but not necessarily the best that either player can do. Consideration of all possible coalitions of players and all possible strategies for such coalitions leads to the conclusion that players I and II can form a mutually most advantageous (in the sense that the maximum of their minimum guaranteed payoff is achieved) coalition

and agree to each play pure strategy 2. Then for player I, $2 \leq V_1 \leq 5$, and for player II, $V_2 = 2$.

From this relatively simple example it is clear that the difficulties of analysis would increase rapidly as n increases. Additionally, we have chosen a game with only two possible pure strategies for each player to illustrate coalition formation. The problems associated with, say a ten-person game with each player having 10 possible pure strategies, would involve 10^{10} possible outcomes with 637 possible coalitions!

Other considerations involved in coalition formation, such as permitting transfer of utility between players in order to form a coalition, are not treated here. The interested reader is referred to Luce and Raiffa, Games and Decisions, chapter seven.

10. A Technique for Estimating an Opponent's Strategy

Suppose we are preparing for forthcoming negotiations with another party. We assume that we can identify all possible strategies that either of us can play and we know our own utilities, associated with the possible outcomes. Our previous considerations have assumed that we also know our opponent's utilities. We now wish to assume that we do not know our opponent's utilities, but can estimate them through knowledge of his value system, intelligence information, etc. Using these estimates we want to formulate the negotiations as a two-person non-zero-sum game and calculate an estimated strategy for the opponent. As the negotiations proceed we will modify the estimated strategy in accordance with his

observed play of the game. This procedure can be viewed as a representation of the "feeling out" phase that usually occurs in actual negotiations.

We formulate the game as follows:

I \ II	1	2
1	(a,x)	(b,y)
2	(c,z)	(d,w)

where we are player I and a,b,c,d are known and x,y,z,w are unknown.

We assume that Player II will find some strategy that will guarantee him some minimum expected payoff $\underline{v}$. Suppose he plays pure strategy 1 with probability p. Then

$$px + (1-p)y = \underline{v} \quad ,$$

$$pz + (1-p)w = \underline{v} \quad .$$

Solving for p and $\underline{v}$ in terms of x,y,z,w gives

$$p = \frac{w-y}{x-y+w-z} \quad , \quad \underline{v} = \frac{xw-yz}{x-y+w-z} \quad .$$

Now suppose that we have made estimates $\hat{x}$, $\hat{y}$, $\hat{z}$, $\hat{w}$ on x, y, z, w, respectively. Then our expectation of Player II's strategy to guarantee some payoff will be

$$\hat{p} = \frac{\hat{w}-\hat{y}}{\hat{x}-\hat{y}+\hat{w}-\hat{z}} \quad .$$

If all of our estimates are changed by a constant c, that is, $x_1 = x + c$, etc., the calculated $\hat{p}_1$ is the same as $\hat{p}$. This can be represented graphically as follows:

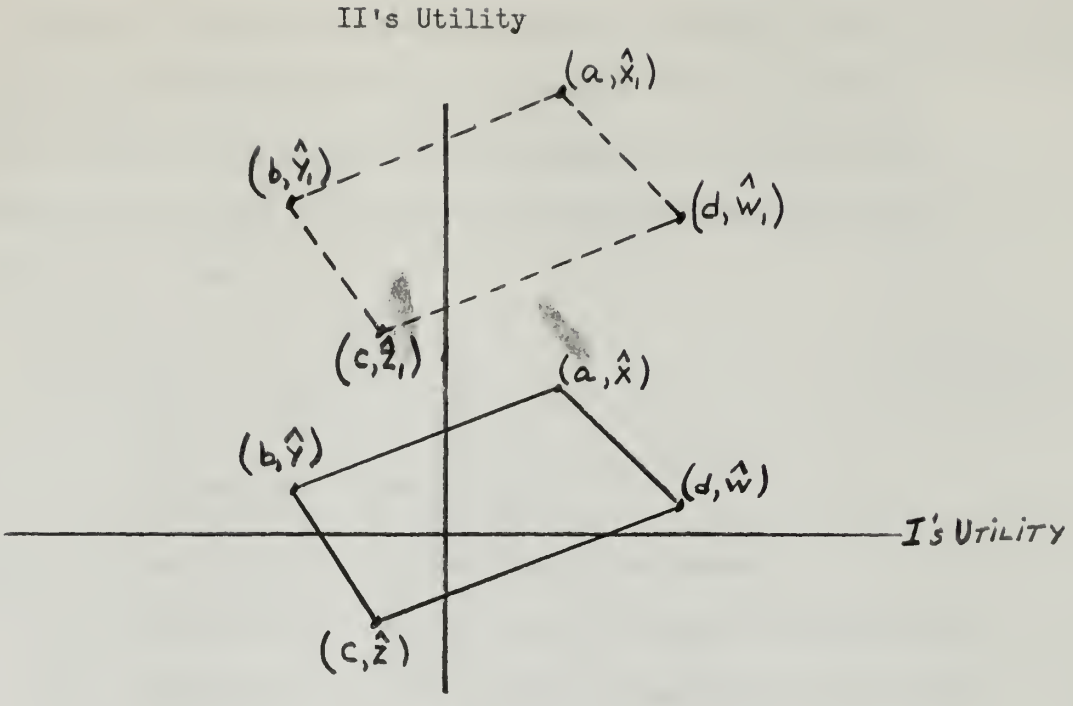


Figure 4

We must have some level of confidence associated with our estimates of x, y, z, w, or we cannot be sure that the game we formulate even vaguely resembles the true situation from observing Player II's plays of the game. Suppose we believe our estimates to be correct within a certain percentage, say $\pm 100\beta$, $0 \leq \beta \leq 1/2$. We can then construct intervals on the utility graph and consider various convex regions.

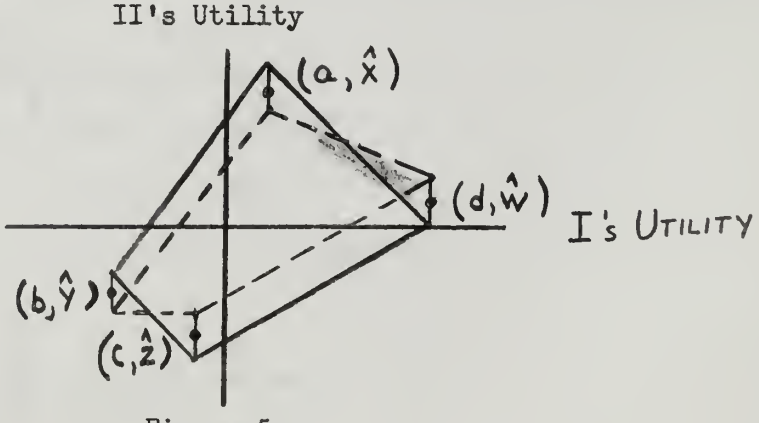
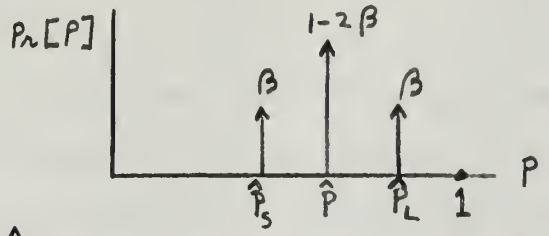


Figure 5

We then choose the largest and smallest convex regions and compute the estimated strategies associated with the utility values defining the corners of the regions. We now have three estimated strategies and assign the following probability distribution to them:



where $\hat{p}$ is the strategy computed from the estimates

$\hat{p}_L$ is the strategy computed from the largest convex region

$\hat{p}_S$ is the strategy computed from the smallest convex region

Now as we observe the pure strategies actually played by Player II we will modify our estimate of Player II's strategy by using Bayes Theorem until $Pr(p = d) \geq \alpha$, $0 \leq \alpha \leq 1$, where α is some level of confidence at which we will act as if Player II's true strategy is to take course of action 1 with probability d and course of action 2 with probability 1 -d.

EXAMPLE 1

Consider the "Battle of the Sexes" game previously discussed:

I \ II	1	2
1	(2,1)	(-1,-1)
2	(-1,-1)	(1,2)

Player I has the following information:

I \ II	1	2
1	(2,x)	(-1,y)
2	(-1,z)	(1,w)

where x, y, z, w are unknown.

Player I reasons:

If I play (2/5, 3/5) I can guarantee myself an expected payoff of 1/5. I have the following estimates:

$$\hat{x} = 3 \quad , \qquad \hat{y} = -2 \quad ,$$

$$\hat{z} = -1 \quad , \qquad \hat{w} = 5 \quad .$$

I am fairly certain that these estimates are correct, at least to 20 percent. If I can obtain a .90 probability on any one strategy of Player II, I will act as if he is actually playing that strategy.

Graphically, the largest convex region is defined by

$$\hat{x}_L = 2.4 \quad , \qquad \hat{y}_L = -2.4 \quad ,$$

$$\hat{z}_L = -0.8 \quad , \qquad \hat{w}_L = 6.0 \quad ,$$

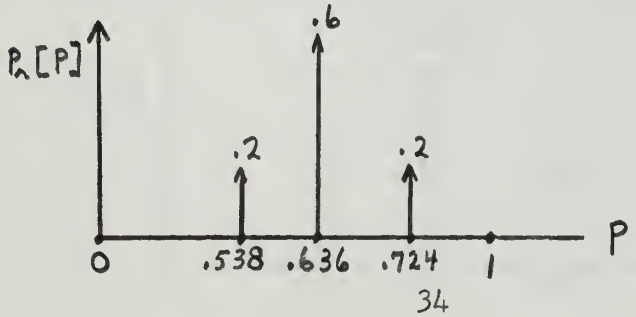
for which $\hat{p}_L = \dfrac{8.4}{11.6} = .724$;

similarly, for the smallest convex region and the original extimates

$$\hat{p}_s = \frac{5.6}{10.4} = .538 \quad ,$$

$$\hat{p} = \frac{7}{11} = .636 \quad .$$

The probability distribution over p is then

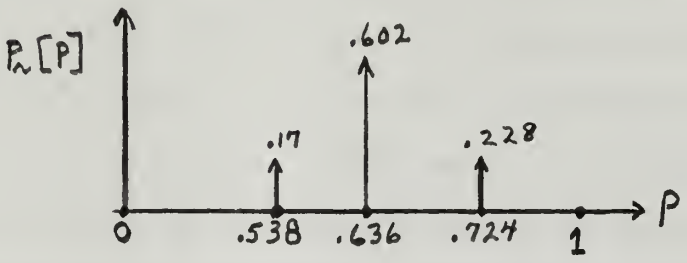


Now suppose that on the first play of the game, Player II uses pure strategy 2. Then the probability distribution is modified to

$$\Pr[p=.636] = \frac{.6(.636)}{.6(.636)+.2(.538)+.2(.724)} = .602$$

$$\Pr[p=.538] = \frac{.2(.538)}{.6(.636)+.2(538)+.2(.724)} = .17$$

$$\Pr[p=.724] = \frac{.2(.724)}{.6(.636)+.2(538)+.2(.724)} = .228$$

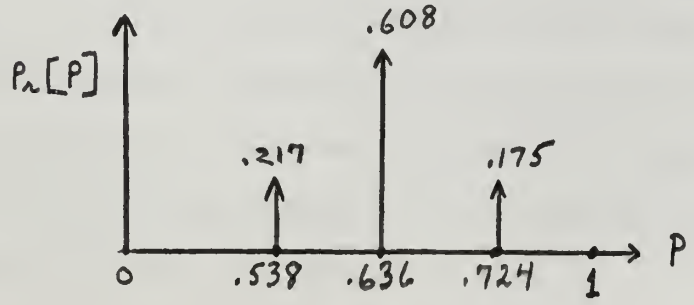


Suppose Player II next plays pure strategy 1. Then

$$\Pr[p=.636] = \frac{.6(.636)(.364)}{.6(.636)+.2(.538)(.462)+.2(.724)(.276)} = .608$$

$$\Pr[p=.538] = \frac{.2(.538)(.462)}{.6(.636)(.364)+.2(.538)(.462)+.2(.724)(.276)} = .217$$

$$\Pr[p=.724] = \frac{.2(.724)(.276)}{.6(.636)(.364)+.2(.538)(.462)+.2(.724)(.276)} = .175$$



This process is continued until $Pr[p=.636] \geq .9$. It is possible that the number of plays of the game may be very large before this result is achieved, or that it may never be achieved. In this case, a significant amount of data will have been obtained. A sensitivity analysis can be performed varying the estimated utilities, the level of confidence associated with the estimated utilities, and the degree of reliability required to act as if Player II is playing a certain strategy.

EXAMPLE 2

There is a Company A engaged in manufacturing at one plant. The labor force, consisting of 100 workers, is represented by Local #718, Union B. The contract between the company and the union is for a period of twelve months and is about to expire. Both management and the union leadership are preparing their positions for the forthcoming renegotiation of the contract. The company has done very well financially during the period of the last contract. Due to its small size Company A can compete with larger companies engaged in the same enterprise principally because of its good reputation for delivering orders on time. At present the company has a backload of orders sufficient to keep the work force busy for a full twelve month period. The wage rate at Company A is higher than other locally prevailing wages, but is lower than the industry standard wage. Consequently, management believes that the principal union demand during negotiation will be an increased hourly wage. In anticipation of this demand, management has calculated that a maximum increase of 9 cents per hour can be granted.

Union B is an established representative of the work force at Company A, having been recognized as the bargaining agent some eight years ago. The union leaders are satisfied with the fringe benefits currently in effect at the company and intend to make an increased hourly wage rate their sole demand in the negotiations. An increase of 16 cents per hour has been set as a goal to be achieved. This would bring the wage rate at Company A into line with the industry standard. If this demand is not met the union is prepared to call a strike. National headquarters of Union B will provide strike funds for a period of six months if a strike takes place.

Payoffs

Under the present wage rate of $2.00 per hour Company A makes a profit of $10,000 per unit sold. One unit per month is the capacity of the plant. If the union members should go on strike the plant will be closed down for one month before it can commence operating again. We make the following definitions:

y = wage increase granted to employees (Dollars per month)

T_1 = number of months employees work

T_2 = number of months employees strike

where $0 \leq T_1 \leq 12, \quad 0 \leq T_2 \leq 12, \quad T_1 + T_2 = 12$

R = rate of production (units per month)

g = profit from sale of one unit under present wage rate (dollars)

Then the profit to Company A for the year following the expiration of the contract will be

$$P_A = RgT_1 - RgT_2 - yT_1$$

$$= Rg\,(T_1 - T_2) - yT_1$$

$$= (20000 - y)\,T_1 - 120000 \quad .$$

If the union goes on strike, the national headquarters will provide $16000 per month in strike fund wages to the work force at Company A. This income for the workers will cease if a strike lasts more than six months. At the present wage rates the workers, working 160 hours a month, have a total payroll of $32000.

The total income to the members of the union over the next year will be

$$P_B = \begin{cases} yT_1 - (32000 - 16000)T_2 & 0 \leq T_2 \leq 6 \\ yT_1 - 32000T_2 - 96000 & 6 \leq T_2 \leq 12 \end{cases}$$

or

$$P_B = \begin{cases} 12y - T_2(y - 16000) & 0 \leq T_2 \leq 6 \\ 12y + 96000 - T_2(y - 32000) & 6 \leq T_2 \leq 12 \end{cases}$$

Thus far the payoffs for the union and the company have been given such that a deterministic point can be computed where both parties can make no gain during the remainder of the contract period. This point is reached when $T_2 = 3.33$. A strike past that length of time would involve net losses to both parties. However, both management and the union will have to consider other, more subjective, aspects of their payoffs. For instance, management might consider that if they fail to de-

liver an order due to a strike the buyer may remove his business to another company permanently. This may cause the loss of future orders. The union should consider the effect of a possible "lost" strike and consequent loss of confidence in the leadership on the part of the members.

Let us assume that management considers an unfilled order as a cost which is equal to twice the profit they would have made on the order. The apparent second loss of profit is to account for possible lost future orders. This factor in management's utility function is unknown to the union. Then the management utility function is

$$U_A = (30000 - y)\, T_1 - 240000 \quad .$$

Further, let us assume that the union members will initially be receptive to a strike in the expectation of future benefits to be derived, but after a strike of two months duration they would prefer to be receiving their wages at work. This factor will be unknown to management. The union leaders consider that this can be crudely represented by including the expression $10000 - 5000T_2$ in their utility function. Then

$$U_B = \begin{cases} 10000 + 12y - T_2(y + 11000) & 0 \leq T_2 \leq 6 \\ 12y + 106000 - T_2(y + 27000) & 6 \leq T_2 \leq 12 \end{cases} \quad .$$

Now suppose that the actual negotiations have started. The union's initial demand is for an 18-3/4 cents per hour increase and management responds with an offer of 6-3/4 cents per hour. If we let pure strategy 1 for each party be agreement on the union demand and pure strategy 2 be agreement on the management offer, the initial positions can be put in a game

matrix:

Union

Management I \ II	1	2
1	(84,000, 36000)	(-27000, -12000)
2	(-27000, -12000)	(108000, 22000)

Payoffs in positions 1,2 and 2,1 are the amounts each party will lose if $T_2 = 1$, i.e., they do not agree and there is a strike for one month. The figues are computed assuming each had accepted the other's initial position.

Now each of the parties can proceed to use example 1 to attempt to determine the other's strategy, with the exception that here the utility functions are not constants. Therefore, the game must be reformulated at intervals of one month.

EXAMPLE 3

Consider example 2, except that we now wish to examine the case where two issues, rather than one, are to be negotiated. Assume that the Union, in addition to the wage increase demanded, has asked for some fringe benefit, such as an unemployment compensation plan.

Suppose the management of Company A has computed that they can afford to grant 5 cents per hour per employee in fringe benefits if the wage increase is held to 9 cents per hour. The union intends to ask for an unemployment compensation plan the cost of which will be 7 cents per hour per employee to the company. Such a plan would provide about 5% of the work force at Company A half pay for each month of unemployment.

At the opening session of the negotiations, the Union demand for fringe benefits is made for the unemployment compensation plan costing the company 7 cents per hour or $1120 per month. Management responds with the offer of a similar plan costing only 5 cents per hour or $800 per month.

We can now formulate the negotiations as a game in two different ways, that is, either as two games involving two separate issues, or simply add the demands and offers and consider the resultant demand and offer as a single game. If we pursue the latter course we obscure the differentiation of the two issues and, in effect, simply reconsider example 2. We have simply added explanation of the issue involved. Therefore, we wish to formulate the negotiations in the first sense, arriving at the following two game matrices:

1. Wages

I \ II	1	2
1	(84000, 36000)	(-27000, -10000)
2	(-27000, -10000)	(108000, 12000)

2. Compensation Plan

I \ II	1	2
1	(-13440, 13440)	(0, 0)
2	(0, 0)	(-9600, 9600)

where player I is Management, Player II is the Union, strategy 1 is to agree to the union demand and strategy 2 is to agree to management's offer.

While there are now two game matrices under consideration it should be kept in mind that they are interdependent, that is, agreement on both issues involved is necessary to a successful conclusion of the negotiations.

The interesting feature of this formulation of the negotiations is the ability of the two parties to transfer utilities from one matrix to the other by making proposals which are trade-offs. For instance, the Union could make the following offer:

> If you (management) will agree to a wage increase of 15 cents per hour along with the compensation plan costing 7 cents an hour, I believe we can reach an agreement.

(Such proposals would reflect the union interest in the issues involved. Here the union believes the acceptance of the compensation plan is more important to the members of the union than the higher wage increase originally demanded.)

Management might respond with an offer to agree to the compensation plan propsed by the union if the wage increase is held to 7 cents an hour. This would still keep management's cost within their goal of no more than 14 cents per hour total cost.

After the proposals have been made, the two game matrices are reformulated as follows:

1. Wages

I \ II	1	2
1	(91200, 28800)	(-28800, -12200)
2	(-28800, -12200)	(105600, 23440)

2. Compensation Plan

I \ II	1	2
1	(-13440, 13440)	(0, 0)
2	(0, 0)	(-13440, 13440)

Here the interdependency of the two game matrices is made even clearer. Apparently, from inspection of the second game matrix, management has agreed to the union compensation plan. This is not the case, however, since the acceptance of the union compensation plan is made contingent upon union acceptance of a certain wage increase as indicated by the first game matrix.

In this example, Management can be concerned solely with the total cost accrued in order to reach a settlement. This is the case because we have chosen the example such that the utilities are directly transferable between the two game matrices. If the second issue raised by the union had been that the union be permitted a voice in affairs within the company, which management considered to be their prerogative, such direct transfer of utility would not be permissible.

11. Conclusion.

Hopefully the reader will carry away two impressions, one general and one specific. The general impression to be conveyed is that the Scientific Method, as practiced by the Operations Analyst, can be fruitfully employed to provide quantitative approximations to certain aspects of non-quantitative problems.

The particular impression to be conveyed is that Game Theory can be a useful technique for providing quantitative insight into certain aspects of the Negotiation Process. The Game-Theoretic conclusions do not purport to be a solution, but they do provide a measure for comparing alternatives. Too often, the authors believe, methodology such as Game Theory, having originally promised much and delivered little in the way of specific solutions, has been dismissed as irrelevant.

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APPENDIX A

Some Properties of Linear Utility Functions

The game theoretic approach to bargaining requires some payoff function which will represent the possible outcomes of the game. The definition and mathematical properties of the payoff function are the subject of Utility Theory.*

In this thesis, we consider only games which have a finite, discrete number of outcomes. Consequently we need only have a finite discrete payoff function and do not consider other functional forms.

Suppose a player, in the game theory sense, with several possible outcomes of the game, say, A,B,C,D, desires to establish relationships among the four outcomes which will assist him in selecting a course of action for the play of the game. For simplicity let us assume that the payoff function assigns real numbers a,b,c,d to the outcomes A,B,C,D, respectively. To obtain the linear utility function we make the following assumptions about the alternatives A,B,C,D:

1. The player considering any two of the outcomes can decide which is preferable or that they are equally desirable.

2. The preferences for outcomes can be ordered; further, the ordering is transitive.

*Utility Theory is not necessarily connected with Game Theory and can be useful in other contexts, but here we are only concerned with its implications for Game Theory.

3. Any probability combination of equally desirable outcomes is just as desirable as either outcome.

4. If outcome A is preferred to B and B is preferred to C, then there is a probability combination of A and C which is just as desirable as B.

5. If p is a probability, $0 \leq p \leq 1$, and outcomes A and B are equally desirable, then pA +(1-p)C and pB +(1-p)C are equally desirable.

These assumptions are sufficient to guarantee a linear utility function.

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